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SOIL MOISTURE CURVE FITTING FOR QUANTIFYING THE HYDRAULIC FUNCTIONS OF UNSATURATED SOILS

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Introduction

The measurement of capillary pressure and the application of soil moisture curve fitting equations for quantifying the hydraulic functions of unsaturated soils is becoming a fundamental part of subsurface investigation. Although the modern technique as developed by van Genuchten (1980, 1991) is almost 10 years old, only recently has it begun to generate a large amount of interest and practical application. Generally, the primary hydraulic function of interest is unsaturated hydraulic conductivity $K_{(\theta)}$ with the second parameter of interest being calculation of relative air permeability. Estimating unsaturated hydraulic conductivity from the measurement of capillary pressure and the application of soil moisture curve fitting equations is a simple two-step process. This paper describes overall procedure including the laboratory technique for measuring a capillary pressure curve (soil moisture retention curve) and the subsequent curve fitting analytical process.

Why quantify unsaturated hydraulic conductivity? Because unsaturated hydraulic conductivity is the basic mode of transport in the vadose zone. Most workers recognize that water is the carrier of a majority of contaminants. When compared to water, air or vapor flow is a minor transport mode even though it can be a major route for volatile organic compounds in vapor phase such as gasoline. With the increased interest in characterization and remediation of subsurface contamination, the ability to accurately characterize subsurface processes is an important step towards realizing a feasible remediation program. This method is also applicable for estimating the relative permeability of Non-Aqueous Phase Liquids (NAPL's, not discussed in this paper).

Methods

There are a wide variety of methods to determine unsaturated hydraulic conductivity including field methods, laboratory methods and calculations from other types of data (Stephens, 1994). Although the methods are numerous, every technique has drawbacks. The easy tests or methods are qualitative and the quantitative tests are expensive and time consuming. This paper focuses on a simple technique that consists of a combination laboratory/mathematical process for determining unsaturated hydraulic conductivity.

There are many field methods and these are often preferred over laboratory methods because they may be more representative of bulk or average properties in heterogeneous soil (Stephens, 1994). Some of the more popular or common field methods are the crust method, sprinkler method, instantaneous profile method, and the disc tension infiltrometer with Guelph permeameter. There are other methods in use although they are not as popular. What most of these methods share in common is that they are often tedious, time consuming, and expensive.

While field methods are often preferred over the laboratory because they may be more representative of bulk or average properties, the laboratory more than makes up for this by providing detailed study and controlled conditions. There is a wider variety of tests available to measure unsaturated hydraulic conductivity in the laboratory and these can be grouped into two categories. Steady-state methods attempt to determine $K_{(\theta)}$ under constant head or constant flux conditions. Numerous tests must be run to obtain conductivities at different moisture contents or potential (Stephens, 1994). Transient methods are used to reduce the time factor required by the experiments though this may be offset by increased test and mathematical complexity. The porous plate methods are not overly complex; their weakness is in the data quality resulting from sample handling techniques inherent to the methodology. There are other methods available but they too are either tedious, time consuming, and/or expensive.

Quantifying unsaturated hydraulic conductivity through the measurement of a capillary pressure curve and the application of soil moisture curve fitting equations has the advantage of being rapid and inexpensive. Determination of $K_{(\theta)}$ from the laboratory curve measurement steps through mathematical analysis can often be completed in a few days. The actual capillary curve measurement itself may be fully automated with no sacrifice in accuracy or precision. Cost is often on the order of a few hundred dollars and this can be brought down significantly through sample volume. And the most attractive part of the method is that it is semi-quantitative. The analytical model and calculations rely on properties (porosity, moisture content, saturated hydraulic conductivity and air permeability) that are measured as part of the sample property/capillary pressure curve determination. The technical procedure consists of four basic steps:

A. Determine sample properties.

B. Measure saturated hydraulic conductivity (K_A) and specific air permeability (K_S).

C. Use centrifuge drainage technique to generate capillary pressure curve (soil moisture retention curve).

D. $K_{(\theta)}$ Calculated by curve fitting and various closed form analytical solutions.

Laboratory Experimental – Determine Capillary Pressure Curve

A. Determine Sample Properties

The process begins with continuous core samples taken in the field. Any core study begins with proper handling of the core with the goal of providing representative undisturbed material. As material from the vadose zone is most often unconsolidated material, proper handling is not only difficult, but also critical. Fortunately, there are advanced techniques for the handling of soft, unconsolidated sediments and selection of the proper core handling facility and geotechnical laboratory should be based on this factor. After coring, the cores are removed from the sampler, labeled, organized for the site geologist and preserved. The core is then transported to the laboratory for processing, sample selection and analysis.

Soft-sediment sampling techniques (see figures 1 and 2) are then used to obtain samples from the core. Sample properties are determined prior to preparation for permeability measurements and centrifuging.

B. Measure Specific Permeability to Water and Air

After basic sample properties are determined, the specific permeability to air is determined. The sample is then vacuum saturated with water and specific permeability to water/hydraulic conductivity at saturation is measured. The sample is ready for loading into sample buckets for centrifuging.

C. Determine Capillary Pressure Curve by Centrifuge (Drainage)

The centrifuge technique for capillary pressure measurement on core samples was introduced by Hassler and Brunner (Forbes, 1997). The centrifuge technique consists in rotating a core at various angular velocities. The core contains two fluids (or one fluid and a gas) for which capillary pressure is to be determined. In the drainage process, the denser fluid (water) is forced out of the core by rotation. Fluid production, or average saturation in the core, is measured at hydrostatic rotation for every speed (See figure 3).

A Beckman Rock/Core Ultracentrifuge is the preferred method for determining capillary pressure curves of core samples and is what was used for this study (see figure 4). The Beckman ultracentrifuge is specially equipped with a viewing port in the chamber door and a strobe light assembly beneath the rotor chamber. A strobe control unit is used to "stop" a bucket directly under the view port so rotor buckets can be visually observed during centrifugation for accurate measurements of liquid volume displaced from the saturated samples. From the correlation of capillary pressure applied and the average saturation measured, complete capillary pressure curves are obtained (see figure 5). Capillary pressure is obtained from the following equation:

$$\Delta \mathrm{P} = 7.94\mathrm{x}10^{-8}(\rho_1 - \rho_2)\mathrm{RPM}^2 r^2$$

Where: ΔP = Capillary Pressure Difference
ρ_1 = Fluid 1 (water) density
ρ_2 = Fluid 2 (air) density
RPM = Rotor revolutions per minute
r = Radial distance from center of axis of rotation to core sample

A drainage curve is measured with water being displaced by air entering the pore space. Eight to twenty pairs of pressure head/moisture data are measured at pressure heads ranging from 0 to 15,000 centimeters (see Table 1). A measurement is complete when the soil moisture content reaches equilibrium with a particular pressure head value. The sample is then removed from the centrifuge and material balance checks are made to verify saturation.

Mathematical Analysis

D. Calculate unsaturated hydraulic conductivity by curve fitting

An analytic function describing the parametric soil moisture/pressure relationship is fit to the capillary data. For these analyses, the van Genuchten (1980) equation was used. Other functions are available (e.g., Brooks-Corey).

van Genuchten Equation

$$\text{Volumetric Moisture, } \theta = \theta_r + \left[\left(\theta_s - \theta_r \right) / \left(1 + \alpha h \right)^n \right]^m$$

Where: θ = Volumetric moisture Content
θ_r = Residual moisture Content
θ_s = Saturated moisture Content
h = Soil Water head
α = van Genuchten Parameter
n = van Genuchten Parameter
m = van Genuchten Parameter

An iterative solver is used to fit the van Genuchten function (1980) to the lab data. The solver is programmed to produce the greatest statistical R^2 value, with values closer to 1.00 indicating a better fit. The curve fits are visually verified. Using the derived capillary parameters, the unsaturated hydraulic conductivity as a function of capillary pressure is calculated using the Mualem expression (1976) and the lab derived value of saturated hydraulic conductivity.

Mualem Equation - Water

$$\text{Krw} = \text{K}_s \text{S}_e^{1/2} \left[\left(1 - \text{S}_e^{1/m} \right)^m \right]^2$$

Where: S_e = Saturation Efficiency (water)
K_s = Saturated Hydraulic Conductivity
m = 1-1/n

Again, using the derived capillary parameters, the effective air permeability as a function of capillary pressure is calculated using the Mualem expression (1976) and the lab derived value of specific air permeability.

Mualem Equation - Air

$$\text{Kra} = \left(1 - \text{S}_t \right)^{1/2} \left(1 - \text{S}_t^{1/\text{m}} \right)^{2\text{m}}$$

Where: S_t = Saturation Efficiency (air)

Output

The capillary pressure curve data is presented in tabular and graphical format (see Table 1 and figure 5). The laboratory data and analytical curve fits are provided (see Table 2 and figure 6). For this exercise actual measured values of water and air permeability were used in the calculations. This is not necessary, the Mualem equations can be calculated using unity values. Unsaturated hydraulic conductivity or relative air permeability can be obtained at any pressure or moisture content by multiplying the corresponding relative permeability scaler by the measured hydraulic conductivity or permeability, respectively.

Practical Uses

Determining the rates for unsaturated hydraulic conductivity in the vadose zone provides workers with an understanding of basic transport modes. Unsaturated hydraulic conductivity can be used to model the retention and movement of water and contaminants in the subsurface. It also gives modelers a handle on the fate and transport of these contaminants. Other uses are design of remediation programs and the scaling of remedial process equipment.

Summary

A procedure has been described for estimating unsaturated hydraulic conductivity through the determination of a capillary pressure curve and the application of soil moisture curve fitting equations. The procedure is rapid, inexpensive and semi-quantitative. The analytical model and calculations rely on measured properties (porosity, moisture content, saturated hydraulic conductivity and air permeability) that are determined as part of the sample property/capillary pressure curve analyses. Use of the method results in simple tabular and graphical output with the user able to obtain unsaturated hydraulic conductivity or relative air permeability at any pressure head, matric potential or moisture content value.

References

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Appendix

Abbreviated Procedure for Estimating Unsaturated Hydraulic Conductivity by Curve Fitting

LABORATORY PROCEDURE – MOISTURE RETENTION CURVE

1) Cut and package core plug using Teflon tape and stainless steel endscreens.
2) Extract sample using a Dean-Stark technique.
3) Dry sample to a stable weight.
4) Determine basic sample properties: grain, pore and bulk volumes. Calculate porosity, grain and bulk densities.
5) Evacuate sample and pressure saturate with filtered and degassed water.
6) Mount core plug into centrifuge cup and Spin at increasing RPMs, monitoring for stability at each setting and recording water volumes produced.
7) Remove sample from centrifuge, Dean-Stark for final water saturation, and dry.
8) Calculate capillary pressure and report basic sample properties.

CALCULATE K_θ - CURVE FITTING

9) An analytical function describing the parametric soil moisture/pressure relationship is fit to the capillary curve data.
10) An iterative solver is used to fit the van Genuchten function to the data
11) The solver is programmed to produce the greatest statistical R2 value, with values closer to 1.00 indicating a better fit.
12) Tabular and graphical output.